Bands and Coulomb effects in ⁵⁰Cr

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Experimental evidence for the coexistence of states with different K^{π} value was found in ⁵⁰Cr. The bandcrossing of the $K=0^+$ ground state band with a $K=10^+$ one is confirmed. Large scale shell model calculations could explain all of the observed experimental features and in particular the known experimental Coulomb energy differences in the mirror pair ⁵⁰Fe-⁵⁰Cr.

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A wealth of experimental information on the structure of nuclei in the middle of the $1f_{7/2}$ shell has been recently collected at LNL [1-6]. The attention was mainly focused on the yrast sequence of states of both natural and unnatural parities up to the band termination in the $1f_{7/2}^n$ and $1d_{3/2}^{-1}$ $\otimes 1f_{7/2}^{n+1}$ configuration space, respectively. In order to investigate rotational collectivity and single particle properties through experimental electromagnetic moments, lifetimes were deduced with the Doppler shift attenuation method (DSAM) for many levels. Large prolate deformation is produced in most ground state (g.s.) bands, which decreases approaching band termination. Large scale shell model (LSSM) calculations for natural parity states were systematically made in the full pf configuration space, getting in general an excellent agreement with the experimental findings [7-9]. The unnatural parity sidebands were described by extending the pf space to include a nucleon-hole in the $1d_{3/2}$ orbital.

In the present work 50 Cr is further investigated [2,3], because of the peculiarity in this region, that the g.s. band shows evidence of bandcrossing, which is mainly based on the presence of two close-lying 10^+ levels. Different interpretations were given for the side band: in Ref. [9] it was suggested to be oblate, while in Ref. [10] to be a high-K prolate one. This question gained more interest recently, because the backbending was shown to be correlated with a discontinuity of Coulomb energy difference (CED) in the mirror pair 50Fe-50Cr [11]. In this context, rotational alignment (RAL), described by the cranked shell model (CSM), was recently proposed [12] as a further explanation of the observed backbending in 50Fe-50Cr, following a previous suggestion [13]. It will be shown that prolate stronglycoupled Nilsson configurations, i.e., deformation alignment (DAL), can explain most of the observed features and that the yrast 10^+ level can be approximately described as a K $=10^{+}$ state, due to the simultaneous excitation of a proton and a neutron from the $[321]3/2^-$ to the $[312]5/2^-$ orbital and from the $[312]5/2^-$ to the $[303]7/2^-$ one, respectively. This was already suggested by the authors in Ref. [14], but additional arguments will be given here.

The reaction ${}^{28}\text{Si}({}^{28}\text{Si},\alpha 2p) {}^{50}\text{Cr}$ was performed at the GASP spectrometer of LNL, using a target of 0.8 mg/cm² backed with 15 mg/cm² of Au, at the bombarding energy of 115 MeV. Experimental data for ${}^{50}\text{Cr}$, obtained from the same experiment [3], are now extended to some nonyrast states. In Fig. 1 the level scheme shows the up-to-date information for the low-lying levels up to the 14⁺ band terminating state in the 1 $f_{7/2}$ space. Only transitions from levels relevant for the present discussion are shown. The levels shown on the leftmost part of the figure were taken from Ref. [15]. The *K* quantum number is assigned on the basis of arguments presented in the following. The experimental properties of positive parity levels of interest are summarized in Table I.

The level at 3324 keV was known to have $I^{\pi}=4^+$ and to decay mostly to the yrast 4^+ state, with a lifetime of $\tau = 0.14(3)$ ps [15].

The level at 3825 keV was reported in Ref. [15] as $(4,5,6)^+$. In the present work an upper limit of 1 ps for its lifetime is estimated by a broad gating on the 541 keV feeding transition from the 5⁻ level at 4366 keV, whose lifetime is determined to be 2.0(5) ps. Both values have been obtained in the present work with a DSAM analysis of the 662 and 541 keV lines, with the procedure described in greater detail in Ref. [3]. From the quoted limit, the squared mixing ratio δ^2 is estimated to be smaller than a few percent. The angular distribution of the 662 keV transition to the 6⁺ state, when gated by the feeding 541 keV one, is forward-peaked as expected for a pure $\Delta I = 0$ *M*1 transition, so that the final assignment to the level at 3286 keV is $I^{\pi} = 6^+$.

The 4⁺ level at 3324 keV is fed by the 3875 keV one with a 551 keV line. The 3875 keV level resulted to have a lifetime of 0.9(3) ps from the analysis of the 712 keV branch so that its decay scheme allows to assign $I^{\pi} = 5^+$.

The level at 3792 keV, having a lifetime of 13(2) ps, has been previously suggested to be 4^{-} [3]. This is in analogy to ⁴⁶Ti, where the yrast 3^{-} level is interpreted as the band head of a $K^{\pi}=3^{-}$ band due to the parallel coupling of a proton in the [202]3/2⁺ Nilsson orbital with one in the [321]3/2⁻. In the present case the first available orbital is the [312]5/2⁻

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FIG. 1. Partial level scheme of ⁵⁰Cr.

and thus a 4⁻ band head is expected. The level at 3792 keV is populated by the 5⁻ level at 4366 keV [15] and decays with equal intensities to the 4⁺ and to the 4⁺₂ level. Both branches have forward peaked angular distributions and are consistent with pure ΔI =1 stretched *E*1 transitions, having a strength of 4×10⁻⁶ and 3×10⁻⁴ W.u., respectively [3]. It must be noted, however, that the angular distributions are also consistent with a 5⁺ level, decaying via two mixed *E*2/*M*1 transitions, having both δ =0.3 [15]. Unfortunately a polarization measurement could hardly make a discrimination since a polarization *P*=-0.4 is predicted with both assignments. Further arguments for a 4⁻ assignment will be provided by LSSM calculations, which are very reliable in this nuclear region.

The comparison of the experimental results with LSSM calculations, performed with the code ANTOINE and using the KB3 interaction, was already presented in Ref. [3], proving the good quality of the wave functions. Detailed theoretical properties for non-g.s. band states are reported in Table I, using the KB3G interaction [16], which gives similar results as using KB3 for this nucleus [3]. A 5⁺ assignment to the 3792 keV level, close to the 5⁺ at 3875 keV, is not realistic since a second 5⁺ level is predicted 450 keV above the yrast

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TABLE I. Transitions in ⁵⁰Cr natural parity sidebands.

| Transition | E_{γ} | E_{γ} | γ-BR | γ-BR | B(E2) | <i>B</i> (<i>M</i> 1) |
|-----------------------------|--------------|--------------|------|------|-----------------------|------------------------|
| | exp. | SM | exp. | th. | th. | th. |
| | (keV) | (keV) | (%) | (%) | $(e^2 \mathrm{fm}^4)$ | (μ_N^2) |
| $K=4^+$ | | | | | | |
| $4_2^+ \rightarrow 2_1^+$ | 2541.0 | 2383 | 0.8 | 0.1 | 0.04 | |
| $4_2^+ \rightarrow 4_1^+$ | 1443.3 | 1308 | 99.2 | 99.9 | 10 | 0.14 |
| $5_1^+ \rightarrow 4_1^+$ | 1993.8 | 1889 | 50 | 56 | 1.0 | 0.01 |
| $5_1^+ \rightarrow 4_2^+$ | 551.0 | 581 | 15 | 8 | 179 | 0.10 |
| $5_1^+ \rightarrow 6_1^+$ | 711.1 | 613 | 35 | 36 | 42 | 0.19 |
| $6_3^+ \rightarrow 5_1^+$ | | 445 | | | 110 | 0.11 |
| $6_3^+ \rightarrow 4_2^+$ | | 1025 | | | 58 | |
| $K = 6^+$ | | | | | | |
| $6_2^+ \rightarrow 4_1^+$ | 1944.4 | 1863 | 13 | 6 | 13 | |
| $6_2^+ \rightarrow 6_1^+$ | 661.5 | 588 | 87 | 94 | 56 | 1.33 |
| $7_1^+ \rightarrow 6_2^+$ | | 1419 | | | 170 | 1.09 |
| $8^+_2 \rightarrow 7^+_1$ | | 740 | | | 150 | 1.91 |
| $8^+_2 \rightarrow 6^+_2$ | | 2158 | | | 42 | |
| $K = 10^{+}$ | | | | | | |
| $11_1^+ \rightarrow 10_1^+$ | 610.1 | 767 | 97 | 95 | 41 | 0.29 |
| $12^+_1 \rightarrow 10^+_1$ | 1272.2 | 1441 | 3 | 4 | 63 | |
| $12_1^+ \rightarrow 11_1^+$ | 662.2 | 674 | 97 | 96 | 48 | 1.48 |

one. This is a further strong support for a 4^- assignment to the 3792 keV level. As a complement to Table I, the calculated lifetime of the 4^+ and 5^+ yrare levels are 0.14 ps and 1.0 ps, in agreement with the experimental values of 0.14(3) and 0.9(3) ps, respectively. The very good general agreement with theory legitimizes relying on predicted, even if not observed, quantities. Thus we note that the calculated large B(E2) values, both for $\Delta I=2$ and $\Delta I=1$, are a reliable monitor of rotational bands even if some configuration mixing has to be considered.

Further arguments for the K-value assignments are given by the calculated electric and magnetic moments reported in Table II. The calculated g factors of the yrare 4^+ and 6^+ levels are 1.33 and -0.12, pointing to their $1f_{7/2}$ proton and neutron pair nature, respectively. In a prolate Nilsson scheme we obtain $K^{\pi} = 4^+$ and 6^+ bands by promoting a proton or a neutron from the $[321]3/2^-$ to the $[312]5/2^-$ orbital and from the $[312]5/2^-$ to the $[303]7/2^-$, respectively, and by coupling the unpaired nucleons to the highest value of K. Besides the calculated static quadruple moment Q_s , the intrinsic electric quadrupole moment Q_0 is reported, which was deduced from Q_s using the standard formula for rotational bands [17]. The intrinsic quadrupole moment was extracted also using the corresponding formula for B(E2) reduced rates [17] and is identified with Q_{t1} and Q_{t2} for transitions with $\Delta I = 1$ and $\Delta I = 2$, respectively. Notice that an intrinsic quadrupole moment of the order of $100 \ e \text{fm}^2$ corresponds to a deformation parameter $\beta \simeq 0.30$. The calculated Q_t value of the g.s. band decreases with the excitation energy in a rather smooth way, when assuming a definite Kvalue. The value Q_0 decreases more rapidly owing to a

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TABLE II. Calculated quadrupole moments and g factors in⁵⁰Cr.

| Level | Ε | Ε | Q_s | Q_0 | Q_{t1} | Q_{t2} | g factor |
|--------------------|-------|-------|---------------------|---------------------|---------------------|---------------------|----------|
| | exp | th | | | | | |
| | (keV) | (keV) | $(e \mathrm{fm}^2)$ | $(e \mathrm{fm}^2)$ | $(e \mathrm{fm}^2)$ | $(e \mathrm{fm}^2)$ | |
| $\overline{K=0^+}$ | | | | | | | |
| 2+ | 783 | 801 | -26.9 | 95 | | | 0.56 |
| 4+ | 1881 | 1876 | -33.6 | 93 | | 98 | 0.75 |
| 6+ | 3163 | 3151 | -20.1 | 50 | | 86 | 0.75 |
| 8 + | 4744 | 4776 | -18.7 | 45 | | 78 | 0.80 |
| 10^{+}_{2} | 6754 | 6859 | 12.3 | | | 48 | 0.59 |
| $K = 10^{+}$ | | | | | | | |
| 10^{+} | 6340 | 6396 | 23.7 | | | | 0.51 |
| 11^{+} | 6950 | 7163 | 27.3 | 49 | 45 | | 0.55 |
| 12^{+} | 7612 | 7837 | 12.7 | 31 | 40 | 187 | 0.58 |
| $K = 4^{+}$ | | | | | | | |
| 4^{+}_{2} | 3324 | 3184 | 32.1 | 63 | | | 1.33 |
| 5 + | 3875 | 3765 | 12.0 | 52 | 74 | | 1.12 |
| 6^+_3 | | 4209 | -14.2 | | 56 | 96 | 0.82 |
| $K = 6^+$ | | | | | | | |
| 6^{+}_{2} | 3825 | 3679 | 43.1 | 69 | | | -0.12 |
| 7+ | | 5158 | 28.3 | 74 | 78 | | 0.07 |
| 8^{+}_{2} | | 5898 | 15.2 | 73 | 65 | 105 | 0.25 |

higher sensitivity to *K*-mixing of the related formula. The *g*-factor values along the g.s. band indicate some prevalence of proton configurations.

Experimental levels are compared with theoretical ones in Fig. 2, according to the *K* assignment. The agreement is very good as only a small offset is observed. One could expect $K^{\pi}=1^+$ bands from an antiparallel coupling. Low spin levels are in fact observed [15], as well as predicted, as reported in Fig. 2, and resemble to a partly decoupled $K^{\pi}=1^+$ band. This confirms the reliability of a strong coupling approach, as already found in ⁴⁹Cr, ⁴⁷V [4,5], and ⁴⁶V [6].

The sudden change of the sign of Q_s at the yrast 10^+ state can be explained by the simultaneous recoupling of a proton pair to $K^{\pi} = 4^+$ and of a neutron pair to $K^{\pi} = 6^+$. For a 10^+ level belonging to the g.s. band, $Q_s = -20 \ efm^2$ is expected, while for the band head of a $K^{\pi} = 10^+$ band with $\beta = 0.25$, $Q_s = 60 \ efm^2$ is predicted. The calculated positive values for both close lying 10^+ levels is explained as due to a strong mixing between the two states. Much smaller mixing is predicted when using the FPD6 [18] or the pairing-plusquadrupole (PPQ) [19] residual interactions, as values of 46, -4 and 47, $-14 \ efm^2$ were, respectively, obtained for the yrast and the yrare 10^+ levels.

In these cases the continuation of the ground band with the yrare 10^+ level is clearly indicated by the negative sign. Moreover, the value of Q_s for the yrast 10^+ state is consistent with a $K^{\pi} = 10^+$ state, having a reduced deformation at such a high spin. This interpretation was already made when discussing lifetime measurements [3], since the yrare 10^+ state decays to the 8^+ state with a B(E2) value of $136(26) e^2 \text{fm}^4$, while the transition from yrast 10^+ state has only about half this strength [3]. In Ref. [3] was already





FIG. 2. Comparison of experimental and theoretical levels in 50 Cr, classified according to quantum number *K*.

argued that with the KB3 residual interaction a too large mixing between the two 10^+ levels is predicted, as a value of 79 e^2 fm⁴ was calculated for the yrare 10^+ level, noticeably lower than the experimental value of 136 e^2 fm⁴. Better agreement with experiment, i.e., 140 e^2 fm⁴ and 145 e^2 fm⁴ is obtained using the FPD6 [18] and the PPQ [19] interactions, respectively. Correspondingly the *B*(*E*2) value is calculated to be smaller for the yrast 10^+ level, as observed, while it has comparable size using the KB3 interaction.

On the other hand a RAL mechanism cannot give a consistent picture of previous data. According to their g factor, the yrare 4^+ and 6^+ levels would be nearly fully aligned. However, rotationally aligned bands based on those states would have essentially $K^{\pi}=0^+$ and thus a quadrupole character and a negative Q_s , in disagreement with LSSM predictions. These arguments can be repeated for the yrast 10^+ level, when interpreted as a rotationally aligned state. Also the interpretation of an oblate yrast 10^+ [9] is not realistic as a low value of K is expected on the oblate side, but states with lower spin of the crossing band were not observed.

The previous conclusions for the 10^+ levels have an important consequence in the interpretation of the CED, which has been shown to be sensitive to the alignment of proton or neutron pairs. In fact, the breaking of a proton pair reduces the spatial overlap between the two protons. This reduces the Coulomb interaction and thus increases the binding energy, while in the mirror nucleus, where a pair of neutrons aligns, the Coulomb energy is not affected. Data are available for odd-*A* mirrors, namely, ${}^{47}\text{Cr}{}^{-47}\text{V}$ [20], ${}^{49}\text{Mn}{}^{-49}\text{Cr}$ [21], ${}^{51}\text{Fe}{}^{-51}\text{Mn}$ [22,23] and for even-even mirror pairs ${}^{46}\text{Cr}{}^{-46}\text{Ti}$



FIG. 3. Experimental CED in (a) 49 Mn- 49 Cr [23], (b) 50 Fe- 50 Cr [12], and (c) 51 Fe- 51 Mn [24,25].

[24] and ⁵⁰Fe-⁵⁰Cr [11]. Experimental data are shown in Fig. 3 for the more relevant cases. LSSM calculations agree well with the experimental CEDs in all of the mentioned mirror pairs, when accounting for the different radius of $1f_{7/2}$ and $2p_{3/2}$ orbitals [25]. A RAL mechanism was proposed about ten years ago for the case of ⁴⁹Cr-⁴⁹Mn [13], in the framework of the CSM. Recently, a more general study was performed on the same line [12]. One should note, however, that CEDs are sensitive to nucleon pair alignment, but cannot discriminate between RAL and DAL descriptions. As previously discussed, the latter is to preferred because it is consistent with most observables, except for some configuration mixing effects.

CED calculations using LSSM wave functions, already reported in Refs. [11,25], have been extended to unobserved nonyrast states and are shown in Fig. 4. In ⁵⁰Fe, the 6_2^+ state is generated by a recoupling of a pair of protons to K=6, with a corresponding reduction in their Coulomb energy at this point. In ⁵⁰Cr, the same effect occurs, but for a pair of



FIG. 4. Theoretical (squares) and experimental (circles) CED $in^{50}\mbox{Fe-}^{50}\mbox{Cr}.$

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neutrons, thus with no Coulomb effects. This gives rise to the large negative CED value for the 6_2^+ state. Similar arguments apply to the 4_2^+ state with the difference that the recoupling of a pair of protons to K=4 occurs now in 50 Cr. This gives rise to a positive CED value for the 4_2^+ state. The calculated CED value for yrast 10^+ state is approximately equal to the sum of the predicted CED's for the 4_2^+ and 6_2^+ states, in agreement with its interpretation as a K=10 state due to the simultaneous alignment of a proton and a neutron pair. One notes, however, that the theoretical discontinuity of CED at the yrast 10^+ is much larger than the experimental one. A better agreement with the experimental finding could be achieved when other effects, such as the drift in nuclear radii with increasing spin, are included in the calculations [25].

A more evident drop in CED occurs in ⁵¹Mn-⁵¹Fe, which can be interpreted with a sudden alignment at $I = 17/2^-$ [Fig. 3(c)]. The $17/2^-$ state is isomeric, suggesting a change of shape, that can be associated with a sort of band crossing. An explanation similar to that in ⁵⁰Fe-⁵⁰Cr can be proposed. As a deformation $\beta \sim 0.22$ is observed at low spin [15], likely due to the action of the $p_{3/2}$ orbital, the yrast $17/2^$ level can be represented as a $K^{\pi} = 17/2^-$ state, produced by promoting one nucleon from the [4321]5/2⁻ orbital to the [330]7/2⁻ and coupling all unpaired nucleons to the maximum *K*.

One notes that the proposed proton pair alignment to $K^{\pi} = 6^+$ for both the yrast 10⁺ state in ⁵⁰Fe and the 17/2⁻ state in ⁵¹Fe agrees with the expectation value of the alignment operator $[(a_{\pi}^+ a_{\pi}^+)^6 (a_{\pi} a_{\pi})^6]^0$ [11,23].

The explanation of CED in ⁴⁹Cr-⁴⁹Mn is probably different. The maximum CED effect is observed at $19/2^-$ and the alignment is smooth. This seems to be correlated with the observed backbending at $19/2^-$, which has been shown in Ref. [4] to not be due to a band crossing. That backbending could be related to the dominance of seniority v = 3 states, as in that case the maximum spin available is just $19/2^-$. This suggestion is reinforced by noting that in ⁴⁸Cr the backbending at $I=12^+$ was explained with the dominance of v=4configurations [26].

Summarizing the present results in ⁵⁰Cr, evidence of two bands with $K^{\pi}=4^+$ and 6^+ was found. The yrast 10^+ level can be approximately described as a $K^{\pi}=10^+$ state. These conclusions were drawn in part via the comparison of very reliable LSSM calculations with the predictions of the strongly coupled Nilsson model. We have, moreover, brought arguments for an interpretation of the alignment mechanisms probed by CED, alternative to that of Ref. [12]. A sudden alignment seems to occur at the bandcrossing with high-*K* bands, while a smoother one is probably related with the seniority scheme, which may compete at high spin with rotational collectivity [26].

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