Apêndice A

Outros sistemas de coordenadas

Os operadores gradiente, divergente e rotacional *dependem do sistema de coordenadas*. A tabela mostra como se apresentam em coordenadas retangulares e também em coordenadas cilíndricas e esféricas. Utilizamos as notações: $\boldsymbol{\nabla} f = \mathrm{grad}\, f$ para o gradiente do campo escalar $f(\mathbf{r})$, $\boldsymbol{\nabla}\cdot\mathbf{A} = \mathrm{div}\,\mathbf{A}$ e $\boldsymbol{\nabla}\times\mathbf{A} = \mathrm{rot}\,\mathbf{A}$ para, respectivamente, o divergente e o rotacional do campo vetorial $\mathbf{A}(\mathbf{r})$.

- **Coordenadas retangulares (x,y,z)**

$$\boldsymbol{\nabla} f = \frac{\partial f}{\partial x}\hat{\mathrm{e}}_x + \frac{\partial f}{\partial y}\hat{\mathrm{e}}_y + \frac{\partial f}{\partial z}\hat{\mathrm{e}}_z$$

$$\boldsymbol{\nabla}\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\boldsymbol{\nabla}\times\mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathrm{e}}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathrm{e}}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathrm{e}}_z$$

- **Coordenadas cilíndricas (ρ,φ,z)**

$$\boldsymbol{\nabla} f = \frac{\partial f}{\partial \rho}\hat{\mathrm{e}}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \varphi}\hat{\mathrm{e}}_\varphi + \frac{\partial f}{\partial z}\hat{\mathrm{e}}_z$$

$$\boldsymbol{\nabla}\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

$$\boldsymbol{\nabla}\times\mathbf{A} = \left(\frac{1}{\rho}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\mathrm{e}}_\rho + \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\mathrm{e}}_\varphi + \frac{1}{\rho}\left(\frac{\partial(\rho A_\varphi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \varphi}\right)\hat{\mathrm{e}}_z$$

- **Coordenadas esféricas (r,θ,φ)**

$$\boldsymbol{\nabla} f = \frac{\partial f}{\partial r}\hat{\mathrm{e}}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\mathrm{e}}_\theta + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial f}{\partial \varphi}\hat{\mathrm{e}}_\varphi$$

$$\boldsymbol{\nabla}\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial(\mathrm{sen}\,\theta A_\theta)}{\partial \theta} + \frac{1}{r\,\mathrm{sen}\,\theta}\frac{\partial A_\varphi}{\partial \varphi}$$

$$\boldsymbol{\nabla}\times\mathbf{A} = \frac{1}{r\,\mathrm{sen}\,\theta}\left[\frac{\partial(\mathrm{sen}\,\theta A_\varphi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \varphi}\right]\hat{\mathrm{e}}_r$$
$$\frac{1}{r}\left[\frac{1}{\mathrm{sen}\,\theta}\frac{\partial A_r}{\partial \varphi} - \frac{\partial(r A_\varphi)}{\partial r}\right]\hat{\mathrm{e}}_\theta + \frac{1}{r}\left[\frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\hat{\mathrm{e}}_\varphi$$
